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Analysis of the Snoek relaxation in Nb–O alloys through the loss tangent distribution function

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Abstract

The Snoek relaxation in an Nb–0.15at.%O alloy reported in the literature, measured at different temperatures and in forced oscillations, is analysed within the framework of the theory of the tangent distribution function and its equivalent form, the integrated distribution, which has been introduced recently in the literature.

It is demonstrated that the relaxation spectrum can be described in terms of a frequency dependence of the pre-exponential factor in the Arrhenius relationship of the relaxation times. A single value for the activation enthalpy is encountered in the whole range of temperatures and frequencies covered by the measurements.

Finally, the results are compared with those previously obtained in an Nb–0.6at.%O alloy as a function of temperature and in free oscillations, where similar values were obtained for the parameters that characterize the relaxation process.

1. Introduction

The internal friction F, which is defined as the ratio of the imaginary part of complex modulus to the real part [1], can be written, as has been demonstrated in recent work [2], in terms of a non-normalized distribution function Ψ_t as

$$F(\omega)=\int_{-\infty}^{\infty} \Psi_t(\ln \tau)\,\frac{\omega\tau}{1+\omega^2\tau^2}\,d(\ln \tau) \qquad (1)$$

where the function Ψ_t, called the tangent distribution function, can be derived from the relaxation spectrum for the moduli or the compliances and the relaxation strength.

The tangent distribution function allows one to include the internal friction in the group of viscoelastic magnitudes which can be expressed by distribution functions, and this enables us, for example, to apply the mathematical procedures of inversion to the internal friction, which could be used, up to now, only for the loss components of the mechanical properties.

Another important advantage of the distribution Ψ_t is the possibility of expressing F in terms of a distribution in the complementary variable of τ, *i.e.* the frequency ω. In fact, the internal friction can be written as [2]

$$F(\omega)=\tilde{\alpha}_t\,\frac{\omega\tau_t(\ln \omega)}{1+\omega^2\tau_t^2(\ln \omega)} \qquad (2)$$

where $\tilde{\alpha}_t=2F_{max}$ and $\tau_t(\ln \omega)$ is called the integrated distribution function. This procedure is an integral form of the mechanical properties, is fully equivalent to the tangent distribution function and provides a method of determining the integrated distribution from a set of internal friction *vs.* frequency curves, each of them characterized by the temperature of the measurement. In fact, the integrated distribution is easily obtained, from eqn. (2), also as function of frequency, for each temperature. It is the purpose of this work to apply the procedure just described to experimental data for the Snoek relaxation in an Nb–0.15at.%O alloy, measured in forced oscillations.

2. Applications

The concepts introduced in this work will be applied to experimental data for the Snoek relaxation, in an Nb–0.15at.%O alloy [3]. The internal friction was measured by using a torsion pendulum operating in forced

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oscillations, with a maximum strain amplitude of 2×10^{-5}. The different symbols in Fig. 1 show the internal friction peaks as function of the oscillation frequency, for different temperatures.

The integrated distribution $\tau_t(\ln\,\omega,\,T)$ can be calculated, for each peak in Fig. 1, by using eqn. (2), with $\bar{\alpha}_t$ given by twice the maximum of each peak. The distributions obtained are a function of both frequency and temperature. However, it is possible to separate the contributions of these magnitudes by proposing a functional form of $\tau_t(\ln\,\omega,\,T)$ as

$$\tau_t(\ln\,\omega,\,T)=\tau_0(\ln\,\omega)\,\exp\left(\frac{H}{kT}\right) \qquad (3)$$

where H is the activation enthalpy of the relaxation process, k is Boltzmann's constant, and $\tau_0(\ln\,\omega)$ is the pre-exponential factor, which contains the whole dependence on frequency. This separation is possible since a plot of $\ln(\tau_t)$ against $1/T$, for different constant frequencies, leads to straight lines, with almost the same slope, as indicated in Fig. 2. It should be pointed out that only some of the experimental data are shown

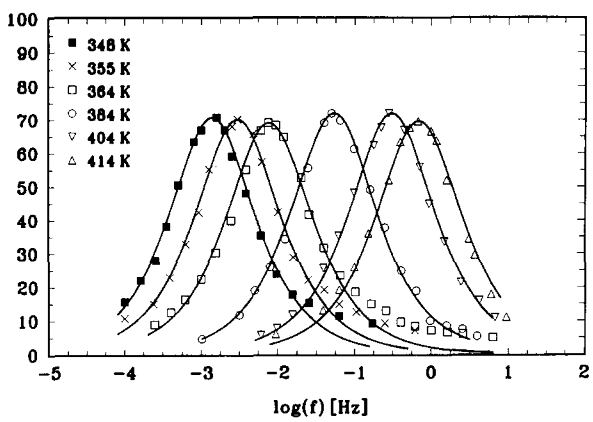

Fig. 1. Internal friction for different temperatures, in an Nb–0.15at.%O alloy [3]. The full curves are obtained by fitting the data to eqn. (2).

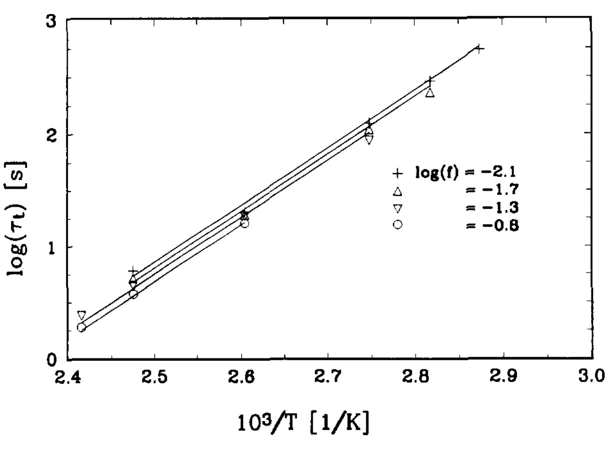

Fig. 2. log τ_t against $1/T$, at different constant frequencies.

to avoid confusion. A least-squares fit to the data leads to the straight lines indicated and to the values of τ_0 and H shown in Table 1. It is evident from this table that H does not change monotonically with frequency but spreads, within more of less 1%, about a mean value

$$H=1.01\pm0.01\ \text{eV} \qquad (4)$$

Once H is known, it is possible to analyse the functional form of the pre-exponential factor in eqn. (3), as a function of frequency, for each peak in Fig. 1. A plot of log τ_0 against log f leads to a straight line at each temperature, indicating a functional form for $\tau_0(\ln\,\omega)$ of the type

$$\tau_0=a\omega^{\alpha} \qquad (5)$$

i.e. a power law in the frequency. The values obtained for a and α, at each temperature, through a least-squares fit to the data, are indicated in Table 2.

A least-squares fit to the data at all temperatures leads to the straight line shown in Fig. 3 and to the values

$$\log a=-12.86\pm0.01 \qquad (6)$$

and

$$\alpha=-0.135\pm0.003 \qquad (7)$$

The full curves in Fig. 1 show the fitting to eqn. (2), by using the parameters given by eqns. (4), (6) and (7).

TABLE 1. τ_0 and H as obtained from the straight lines in Fig. 2

log f (s^{-1})	log τ_0 (s^{-1})	H (eV)
−2.1	−11.87±0.02	1.00±0.01
−1.7	−11.94±0.01	1.010±0.005
−1.3	−11.99±0.01	1.009±0.006
−0.8	−12.06±0.02	1.013±0.003

TABLE 2. Parameters for the power law dependence of the pre-exponential factor at each temperature

Temperature (K)	log a	α
348	−13.0±0.6	−0.14±0.01
355	−12.9±0.1	−0.15±0.05
364	−12.8±0.1	−0.16±0.06
384	−12.99±0.01	−0.131±0.006
404	−12.82±0.01	−0.143±0.002
414	−12.901±0.002	−0.137±0.003

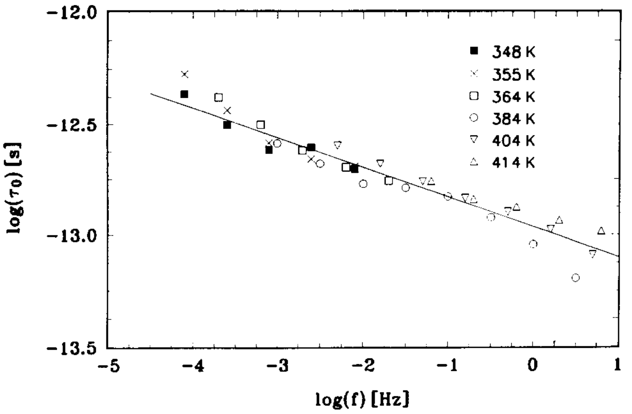

Fig. 3. log τ_0 against log f, for the different peaks in Fig. 1.

3. Discussion

The functional form proposed for the internal friction, plotted in Fig. 1, fits the experimental data quite well, except for the right-hand tails of the peaks, which might indicate the presence of a background. Nevertheless, these results show that there is a non-singular distribution function for the Snoek relaxation process, which can be attributed only to the pre-exponential factor of the Arrhenius representation of τ_t. In fact, the activation enthalpy spreads, within 1%, about an average value, showing no clear tendency with frequency or temperature. The changes in H, indicated in Table 1, affect much less the changes in τ_t generated by the variations produced by the frequency dependence of τ_0. In other words, a distribution in H must be excluded. In fact, it might be assumed that all the data in Fig. 2 can be represented by only one straight line. In this condition, a least-squares fit to the data leads to

$$\log \tau_0 = -12.0 \pm 0.3 \tag{8}$$

and

$$H = 1.01 \pm 0.01 \tag{9}$$

An attempt to fit the data to eqn. (2) with the values of τ_0 and H given by eqns. (8) and (9) leads to the results indicated by the full curves in Fig. 4. It is clear that the experimental values cannot be described in this way.

In summary, it may be concluded that the Snoek relaxation in an Nb–0.15at.%O alloy can be described by a distribution function which affects the pre-exponential factor of the Arrhenius temperature dependence of the relaxation times.

The value for H given in this work is very close to the average value obtained in Nb–0.6at.%O alloys [2], *i.e.* $H = 0.98$ eV, where the internal friction was measured in a traditional torsion pendulum, as a function of temperature and for different moments of inertia (frequencies). Even when the oscillation frequency was

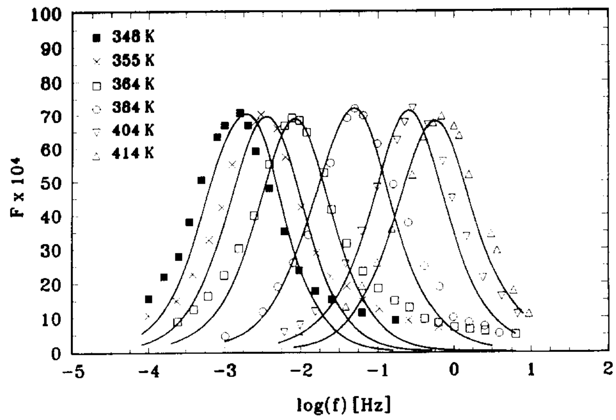

Fig. 4. Fitting of the data to eqn. (2) with τ_0 and H given by eqns. (8) and (9).

TABLE 3. Parameters for the Snoek relaxation in Nb–O alloys obtained by free decay (constant frequency) and in forced oscillations (constant temperature)

Alloy	Experimental procedure	H (eV)	log a	α
Nb–0.15at.%O	Forced oscillations, T = constant	1.01	−12.86	−0.135
Nb–0.6at.%O	Free decay, $f \approx$ constant	0.98	−12.43	−0.125

changed by only an order of magnitude and only three frequencies were involved, the average values

$$\log a = -12.43 \pm 0.08 \tag{10}$$

and

$$\alpha = -0.125 \pm 0.05 \tag{11}$$

were obtained by a least-squares fit to eqn. (5). The parameters obtained in the two alloys and with the two experimental techniques are given in Table 3. It is clear that the results are quite similar and probably a distribution only in τ_0 is involved in the Nb–O alloy. Therefore further work is needed to study the influence of the oxygen content on the parameters of the distribution.

It should be pointed out that the formalism developed in the paper does not make any assumption about the distribution function. Indeed, the fact that the distribution is in the pre-exponential factor and not in the activation enthalpy comes directly from the experimental data.

Finally, it is important to note that the value of H reported in the original work [3] for the Nb–0.15at.%O alloy, *i.e.* $H = 1.16$ eV, is quite different from that obtained in this work. This is mainly because this value for the activation enthalpy was obtained through the shift in the internal friction maxima with temperature, a procedure valid only for a Debye relaxation process.

It is clear that further theoretical and experimental work is needed for a complete understanding of the influence of the tangent distribution function on the internal friction peaks and on the physical meaning of the parameters involved.

4. Conclusions

It has been shown how the integrated distribution for the internal friction, introduced recently in the literature, can be used to interpret internal friction data, measured either as a function of frequency, at different temperatures, or as a function of temperature, at different frequencies. In fact, equivalent parameters for the integrated distribution are obtained in Nb–O alloys, for internal friction measurements performed with two experimental techniques.

Acknowledgments

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References

1 A.S. Nowick and B.S. Berry, *Anelastic Relaxation in Crystalline Solids*, Academic Press, New York, 1972.
2 F. Povolo and C.L. Matteo, *Mater. Trans. Jpn. Inst. Met., 33* (1992) 824.
3 J. Woirgard, *Scr. Metall., 11* (1977) 641.